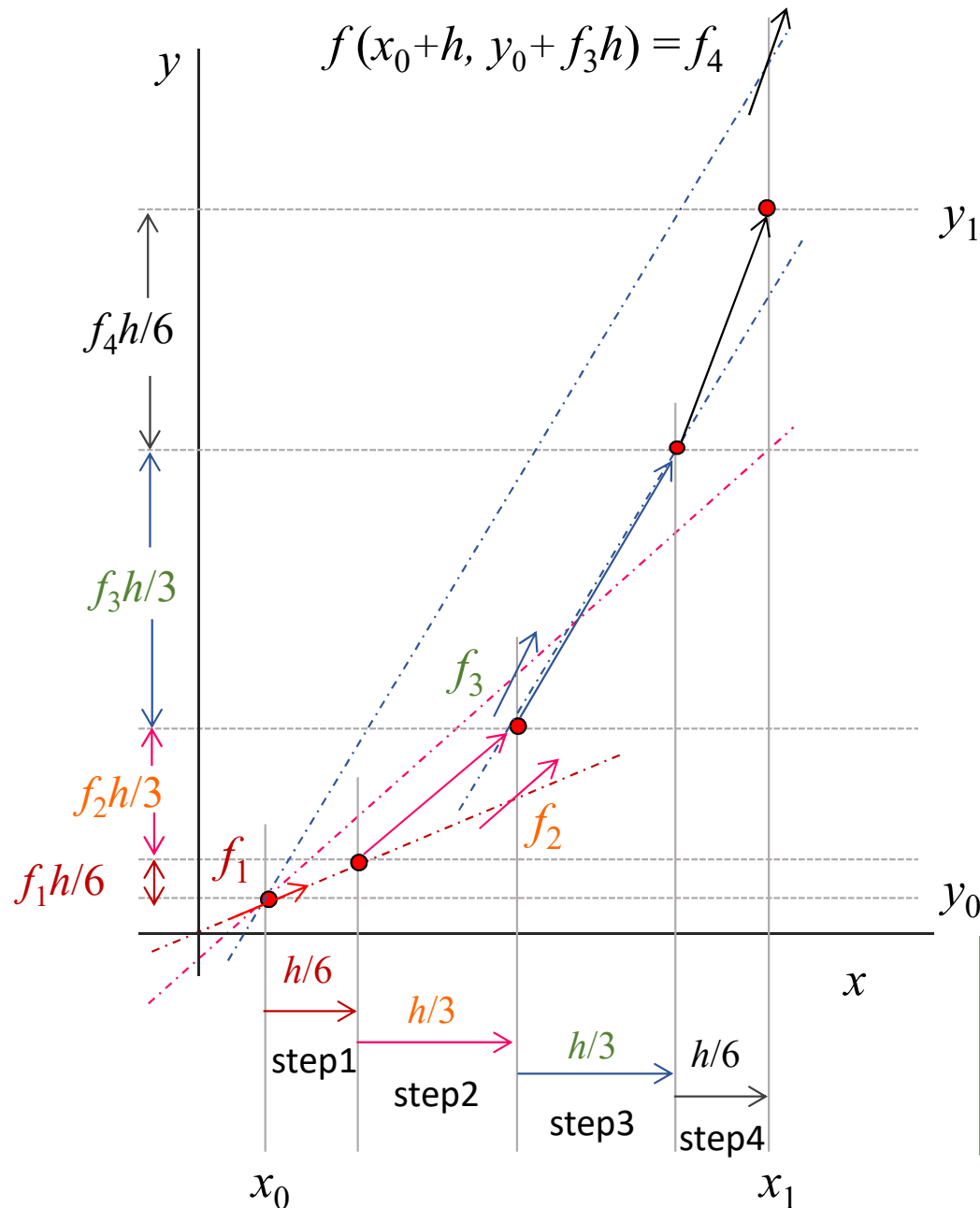


微分方程式の解法 3

$$\frac{dy}{dx} = f(x, y) \text{ をルンゲ・クッタ法で解く}$$



$$\begin{aligned} f(x_0, y_0) &= f_1 \\ f(x_0 + h/2, y_0 + f_1h/2) &= f_2 \\ f(x_0 + h/2, y_0 + f_2h/2) &= f_3 \\ f(x_0 + h, y_0 + f_3h) &= f_4 \end{aligned}$$

$$y_1 = y_0 + f_1h/6 + f_2h/3 + f_3h/3 + f_4h/6$$

$$x_1 = x_0 + h$$

$f(x, y)$ に y 依存性がない場合 シンプソン法に一致

$$y_1 = y_0 + [f(x_0) + 4f((x_0 + x_1)/2) + f(x_1)] h/6$$

微分方程式の解法

ルンゲ・クッタ法による求め方

例 $\frac{d}{dx} y(x) = -y(x)$

$$f(x, y) = -y = \frac{dy}{dx}$$

```
parameter (N=100)
real*8 :: y(N), s=0.d0, h, f, PI, f1, f2, f3, f4
integer :: i
```

```
h=8.d0/N
```

```
y(1)=1.d0 ← 初期条件 y(x=0) = 1
```

```
do i=1, N-1
```

```
  f1=-y(i)
```

```
  f2=-(y(i)+f1*h/2.d0)
```

```
  f3=-(y(i)+f2*h/2.d0)
```

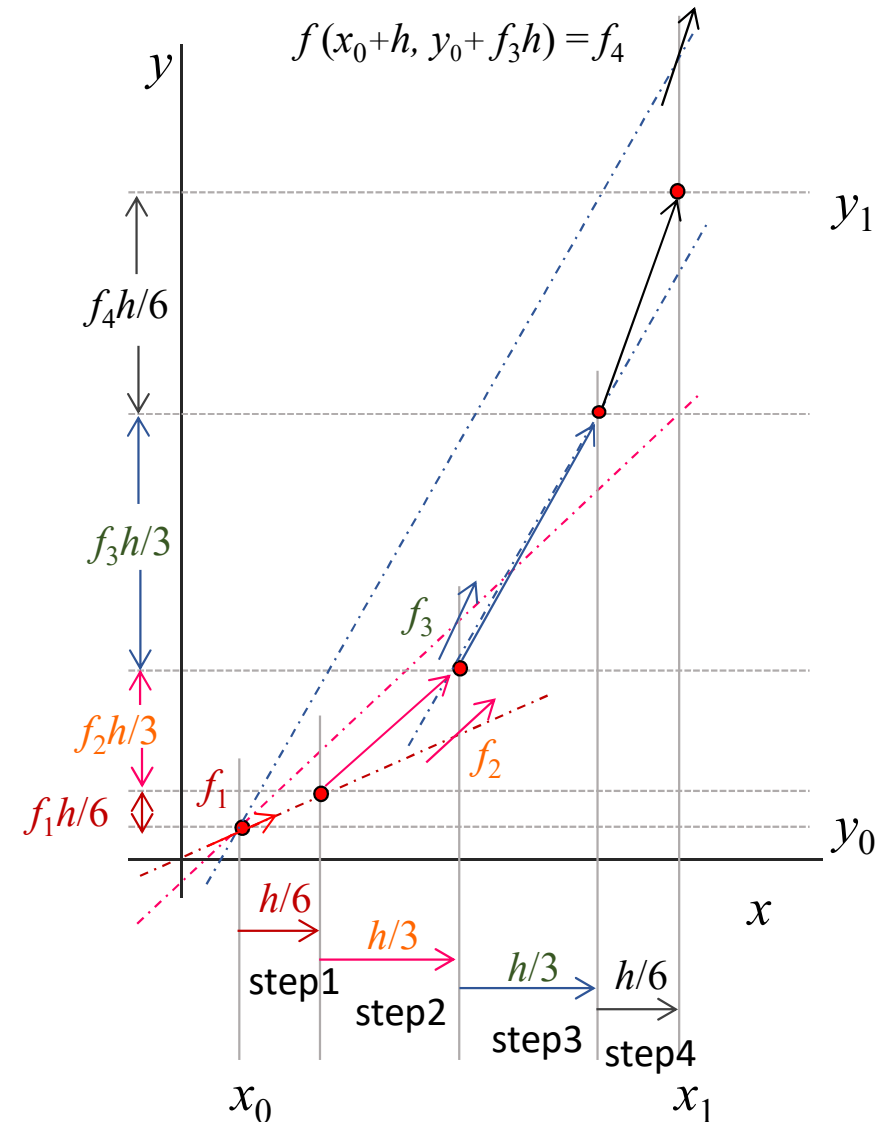
```
  f4=-(y(i)+f3*h)
```

```
  y(i+1)=y(i)+(f1+2*f2+2*f3+f4)*h/6.d0
```

```
  write(*, '(3f12.8)') i*h, y(i+1), exp(-i*h)
```

```
end do
```

```
end
```



微分方程式の解法 4

2 階の微分方程式の解法

$$\frac{d^2 y}{dx^2} = F\left(x, y, \frac{dy}{dx}\right) \quad \rightarrow$$

1 階の連立微分方程式に変換

$$\frac{dy}{dx} = z, \quad \frac{dz}{dx} = F(x, y, z)$$

$$F(x_0, y_0, z_0) = F_1$$

$$F(x_0 + h/2, y_0 + f_1 h/2, z_0 + F_1 h/2) = F_2$$

$$F(x_0 + h/2, y_0 + f_2 h/2, z_0 + F_2 h/2) = F_3$$

$$F(x_0 + h, y_0 + f_3 h, z_0 + F_3 h) = F_4$$

2 階微分の予想値

$$z_0 = f_1$$

$$z_0 + F_1 h/2 = f_2$$

$$z_0 + F_2 h/2 = f_3$$

$$z_0 + F_3 h = f_4$$

1 階微分の予想値

$$z_1 = z_0 + F_1 h/6 + F_2 h/3 + F_3 h/3 + F_4 h/6$$

$$y_1 = y_0 + f_1 h/6 + f_2 h/3 + f_3 h/3 + f_4 h/6$$

具体例

$$\frac{d^2 y}{dx^2} = -y$$



$$-(y_0 + f_2 h/2) = F_3$$

$$\frac{d^2 y}{dx^2} = -y - \frac{dy}{dx}$$



$$-(y_0 + f_2 h/2) - (z_0 + F_2 h/2) = F_3$$

2 階の微分方程式の解法

ルンゲ・クッタ法による求め方

例 $\frac{d^2}{dx^2} y(x) = -y(x)$ $\frac{dy}{dx} = z$, $\frac{dz}{dx} = -y$

```
parameter (N=100)
real*8 :: y(N),z(N),h,PI,f1,f2,f3,f4,g1,g2,g3,g4
integer :: i
```

```
PI=4.d0*atan(1.d0)
```

```
h=2*PI/N
```

```
y(1)=1.d0      初期条件 y(x=0) = 1
```

```
z(1)=0.d0      初期条件 z(x=0) = 0
```

```
write(*,'(4f12.8)') 0*h, y(1), cos(-(0)*h)
```

```
do i=1, N-1
```

```
g1=-y(i)      ← 2 階微分の予想値
```

```
f1=z(i)      ← 1 階微分の予想値
```

```
g2=-(y(i)+f1*h/2.d0)
```

```
f2=(z(i)+g1*h/2.d0)
```

```
g3=-(y(i)+f2*h/2.d0)
```

```
f3=(z(i)+g2*h/2.d0)
```

```
g4=-(y(i)+f3*h)
```

```
f4=(z(i)+g3*h)
```

```
z(i+1)=z(i)+(g1+2*g2+2*g3+g4)*h/6.d0
```

```
y(i+1)=y(i)+(f1+2*f2+2*f3+f4)*h/6.d0
```

```
write(*,'(4f12.8)') i*h, y(i+1), cos(-i*h) , z(i+1)
```

```
end do
```