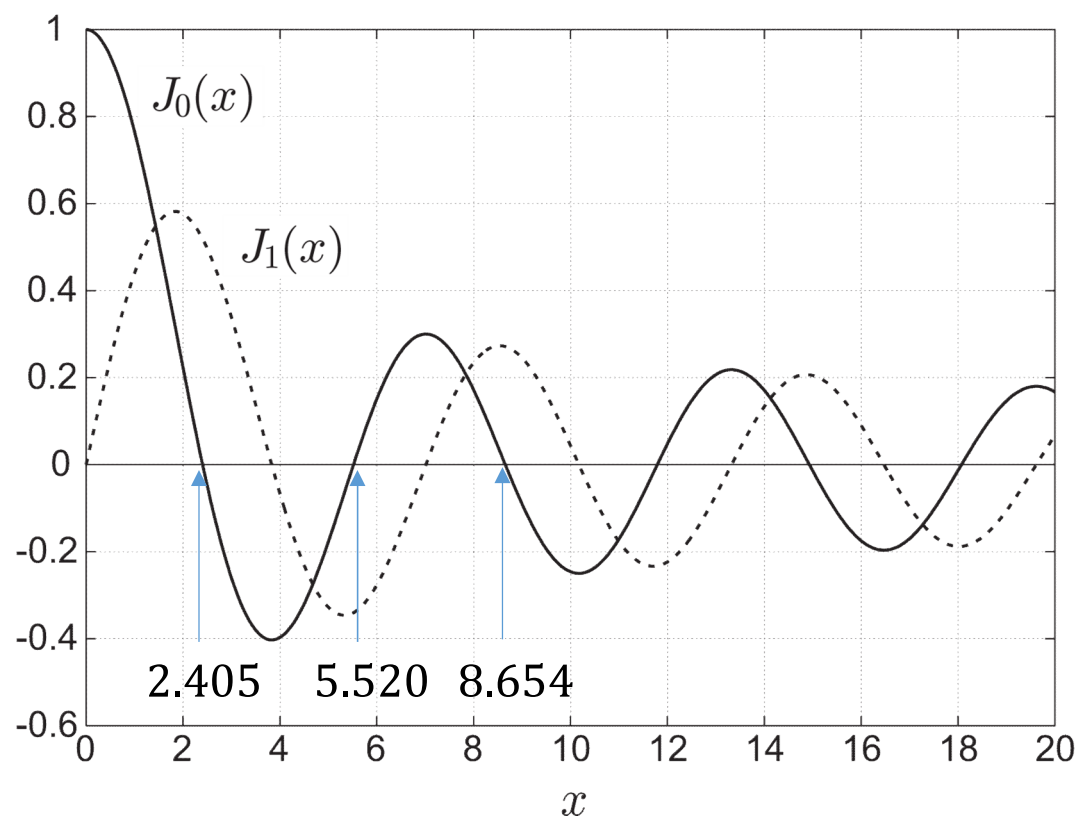


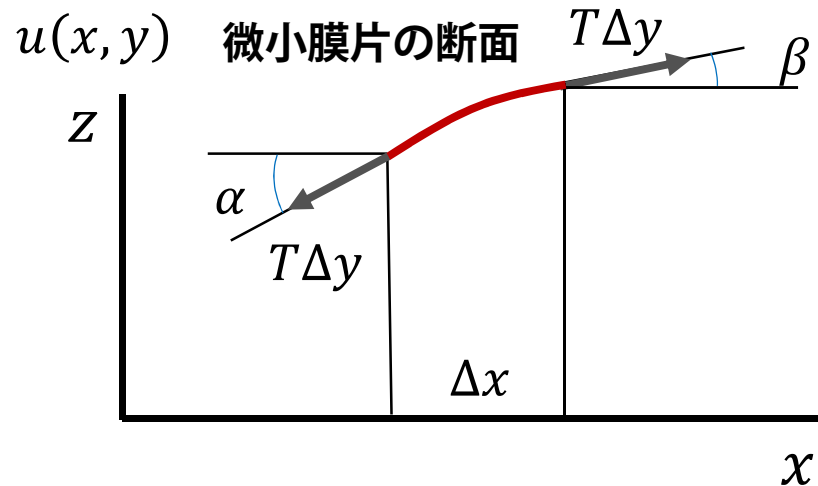
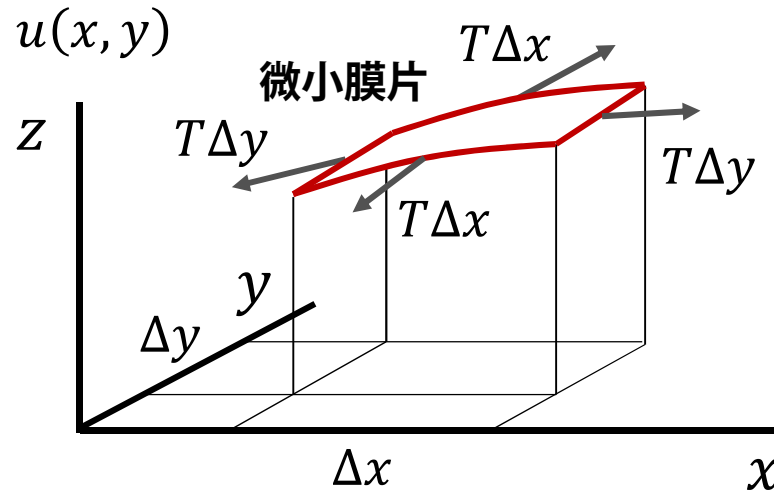
ベッセル関数

$$J_0(x) = 1 - \frac{1}{2^2(1!)^2} x^2 + \frac{1}{2^4(2!)^2} x^4 - \frac{1}{2^6(3!)^2} x^6 + \frac{1}{2^8(4!)^2} x^8 - \dots$$

$$J_1(x) = \frac{1}{2}x - \frac{1}{2^3 1! 2!} x^3 + \frac{1}{2^5 2! 3!} x^5 - \frac{1}{2^7 3! 4!} x^7 + \frac{1}{2^9 4! 5!} x^9 - \dots$$



円形膜(太鼓)の振動



z 方向に働く力: F_z

Δy の辺: $T\Delta y \sin \beta - T\Delta y \sin \alpha$

$$= T \left[\frac{\partial u(x + \Delta x, y)}{\partial x} - \frac{\partial u(x, y)}{\partial x} \right] \Delta y$$

Δx の辺: $T \left[\frac{\partial u(x, y + \Delta y)}{\partial y} - \frac{\partial u(x, y)}{\partial y} \right] \Delta x$

$$\sin \alpha = \tan \alpha = \frac{\partial u(x, y)}{\partial x}$$

$$\sin \beta = \tan \beta = \frac{\partial u(x + \Delta x, y)}{\partial x}$$

円形膜(太鼓)の振動

z 方向に働く力

$$F_z = T \left[\frac{\partial u(x + \Delta x, y)}{\partial x} - \frac{\partial u(x, y)}{\partial x} \right] \Delta y + T \left[\frac{\partial u(x, y + \Delta y)}{\partial y} - \frac{\partial u(x, y)}{\partial y} \right] \Delta x$$

z 方向の微小膜片の運動方程式

$$\rho \Delta x \Delta y \frac{\partial^2 u(x, y)}{\partial t^2} = F_z = T \left[\frac{\partial u(x + \Delta x, y)}{\partial x} - \frac{\partial u(x, y)}{\partial x} \right] \Delta y + T \left[\frac{\partial u(x, y + \Delta y)}{\partial y} - \frac{\partial u(x, y)}{\partial y} \right] \Delta x$$

$$\frac{\rho}{T} \frac{\partial^2 u(x, y)}{\partial t^2} = \frac{\left[\frac{\partial u(x + \Delta x, y)}{\partial x} - \frac{\partial u(x, y)}{\partial x} \right]}{\Delta x} + \frac{\left[\frac{\partial u(x, y + \Delta y)}{\partial y} - \frac{\partial u(x, y)}{\partial y} \right]}{\Delta y}$$

膜片の運動方程式

$$\frac{\rho}{T} \frac{\partial^2 u(x, y)}{\partial t^2} = \frac{\partial^2 u(x, y)}{\partial x^2} + \frac{\partial^2 u(x, y)}{\partial y^2}$$

円形膜(太鼓)の振動

$$\frac{\rho}{T} \frac{\partial^2 u(x, y)}{\partial t^2} = \frac{\partial^2 u(x, y)}{\partial x^2} + \frac{\partial^2 u(x, y)}{\partial y^2}$$

$$u(x, y) = u(x, y, t) = U(x, y)P(t)$$

$$\frac{\rho}{T} \frac{1}{P(t)} \frac{\partial^2 P(t)}{\partial t^2} = \frac{1}{U(x, y)} \left[\frac{\partial^2 U(x, y)}{\partial x^2} + \frac{\partial^2 U(x, y)}{\partial y^2} \right] = -\lambda$$

$$\frac{\rho}{T} \frac{\partial^2 P(t)}{\partial t^2} = -\lambda P(t) \quad \frac{\partial^2 U(x, y)}{\partial x^2} + \frac{\partial^2 U(x, y)}{\partial y^2} = -\lambda U(x, y)$$

$$P(t) = \exp(i\omega t)$$

$$-\omega^2 \frac{\rho}{T} \exp(i\omega t) = -\lambda \exp(i\omega t)$$

$$\omega^2 \frac{\rho}{T} = \lambda \quad \omega = \pm \sqrt{\lambda \frac{T}{\rho}}$$

円形膜(太鼓)の振動

$$\frac{\partial^2 U}{\partial r^2} + \frac{1}{r} \frac{\partial U}{\partial r} + \frac{1}{r^2} \frac{\partial^2 U}{\partial \theta^2} = -\lambda U$$

$$U(r, \theta) = R(r)\Phi(\theta)$$

$$\frac{r^2}{R(r)} \left(\frac{\partial^2 R(r)}{\partial r^2} + \frac{1}{r} \frac{\partial R(r)}{\partial r} \right) + \lambda r^2 = -\frac{1}{\Phi(\theta)} \frac{\partial^2 \Phi(\theta)}{\partial \theta^2} = \beta$$

$$\frac{\partial^2 R(r)}{\partial r^2} + \frac{1}{r} \frac{\partial R(r)}{\partial r} + \lambda = \frac{\beta}{r^2} R(r) \quad x = r\sqrt{\lambda}$$

$$\frac{\partial^2 y(x)}{\partial x^2} + \frac{1}{x} \frac{\partial y(x)}{\partial x} + \left(1 - \frac{n^2}{x^2} \right) y(x) = 0$$

$$\frac{\partial^2 \Phi(\theta)}{\partial \theta^2} = -\beta \Phi(\theta)$$

$$\Phi(\theta) = \cos n\theta, \sin n\theta$$

$$\Phi(\theta + 2\pi) = \Phi(\theta)$$

$$\beta = n^2$$

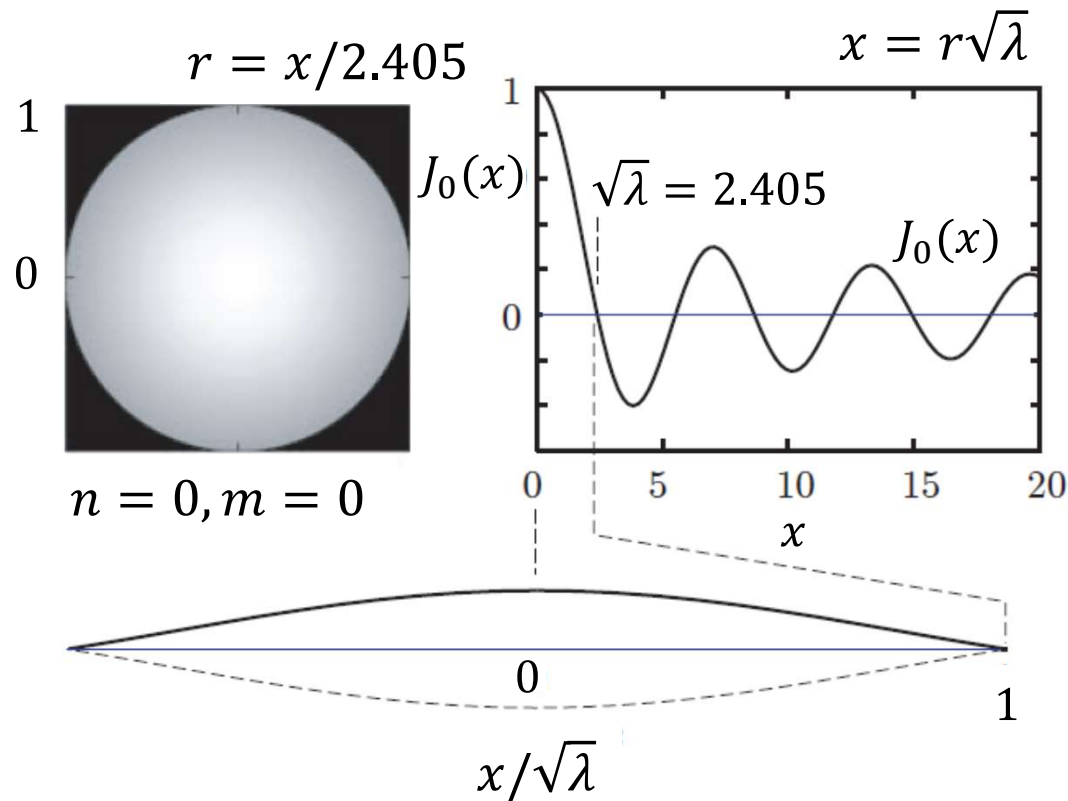
円形膜(太鼓)の振動

テキスト: p. 37

$$n = 0$$

$$\frac{\partial^2 y(x)}{\partial x^2} + \frac{1}{x} \frac{\partial y(x)}{\partial x} + \left(1 - \frac{n^2}{x^2}\right) y(x) = 0$$

$$\Phi(\theta) = \cos n\theta, \sin n\theta$$



角振動数

$$\omega = \pm \sqrt{\lambda \frac{T}{\rho}}$$

$$= \pm 2.405 \sqrt{\frac{T}{\rho}}$$

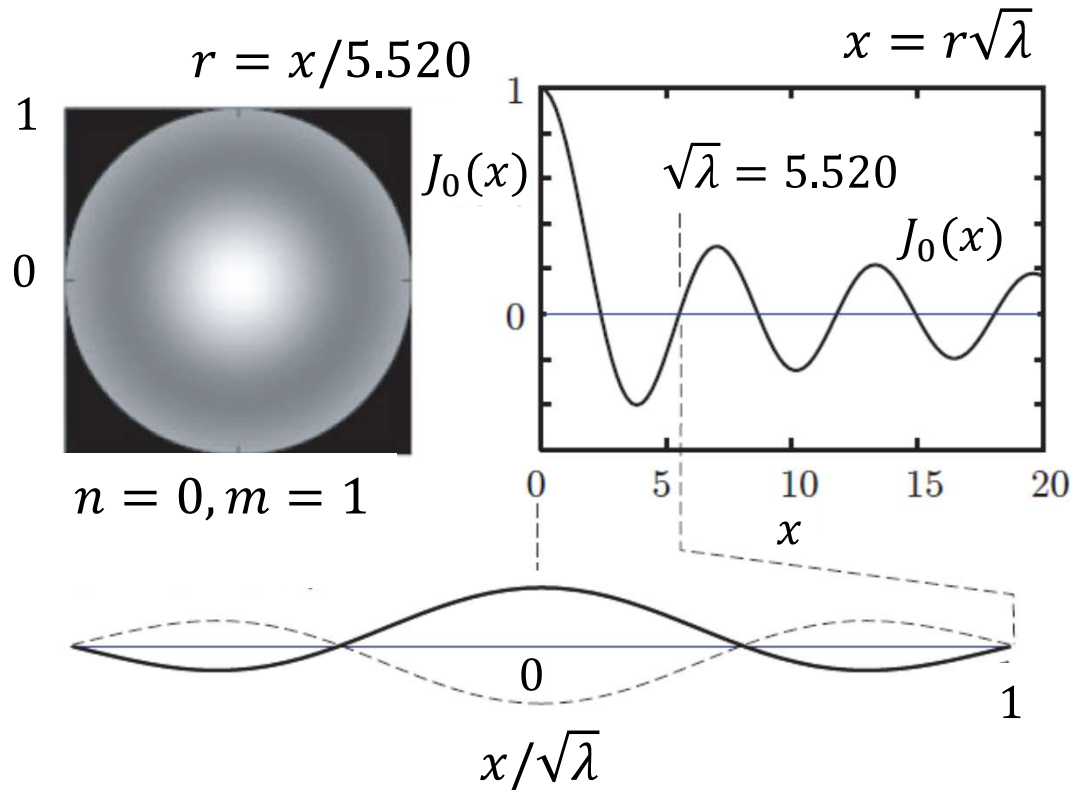
円形膜(太鼓)の振動

テキスト: p. 37

$$n = 0$$

$$\frac{\partial^2 y(x)}{\partial x^2} + \frac{1}{x} \frac{\partial y(x)}{\partial x} + \left(1 - \frac{n^2}{x^2}\right) y(x) = 0$$

$$\Phi(\theta) = \cos n\theta, \sin n\theta$$



角振動数

$$\omega = \pm \sqrt{\lambda \frac{T}{\rho}}$$

$$= \pm 5.520 \sqrt{\frac{T}{\rho}}$$

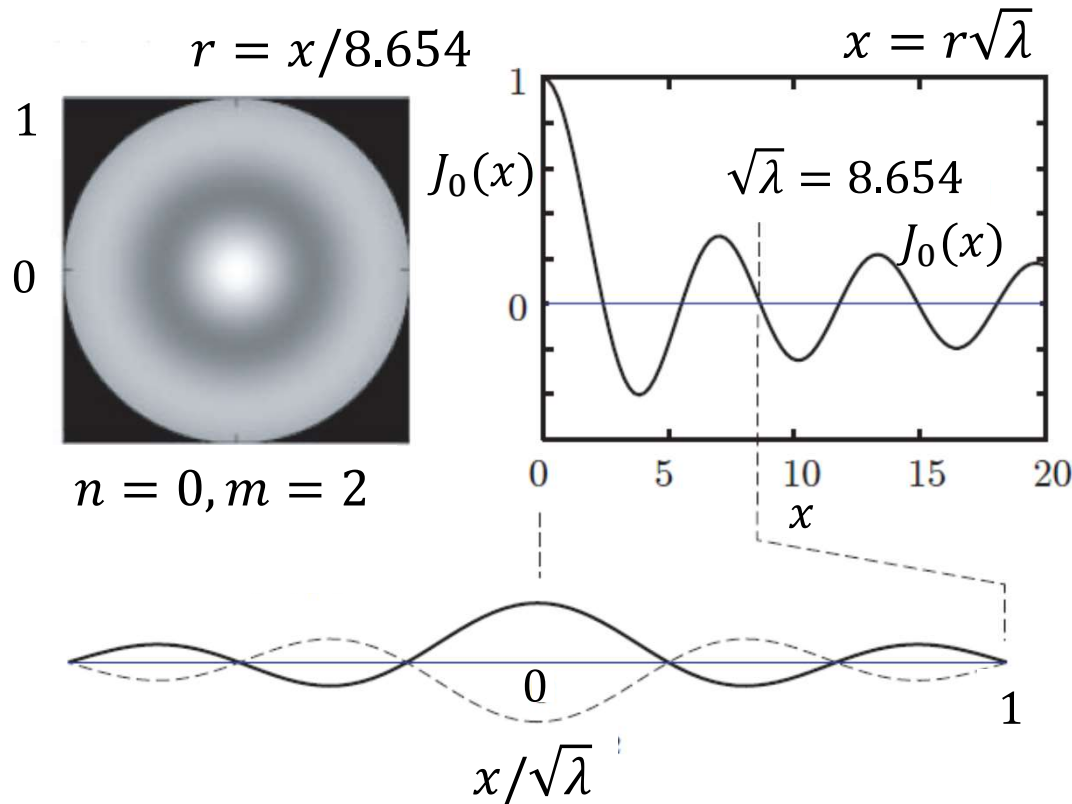
円形膜(太鼓)の振動

テキスト: p. 37

$$n = 0$$

$$\frac{\partial^2 y(x)}{\partial x^2} + \frac{1}{x} \frac{\partial y(x)}{\partial x} + \left(1 - \frac{n^2}{x^2}\right) y(x) = 0$$

$$\Phi(\theta) = \cos n\theta, \sin n\theta$$



角振動数

$$\omega = \pm \sqrt{\lambda \frac{T}{\rho}}$$

$$= \pm 8.654 \sqrt{\frac{T}{\rho}}$$

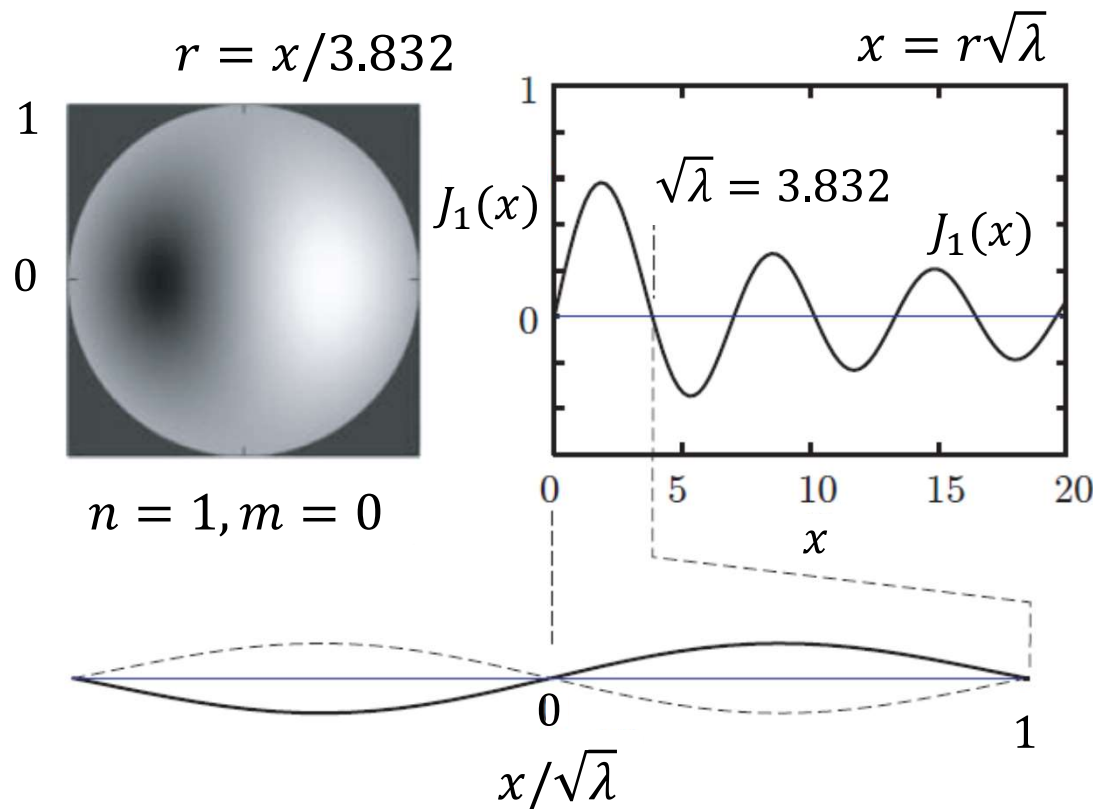
円形膜(太鼓)の振動

テキスト: p. 37

$$n = 1$$

$$\frac{\partial^2 y(x)}{\partial x^2} + \frac{1}{x} \frac{\partial y(x)}{\partial x} + \left(1 - \frac{n^2}{x^2}\right) y(x) = 0$$

$$\Phi(\theta) = \cos n\theta, \sin n\theta$$



角振動数

$$\omega = \pm \sqrt{\lambda \frac{T}{\rho}}$$

$$= \pm 3.832 \sqrt{\frac{T}{\rho}}$$

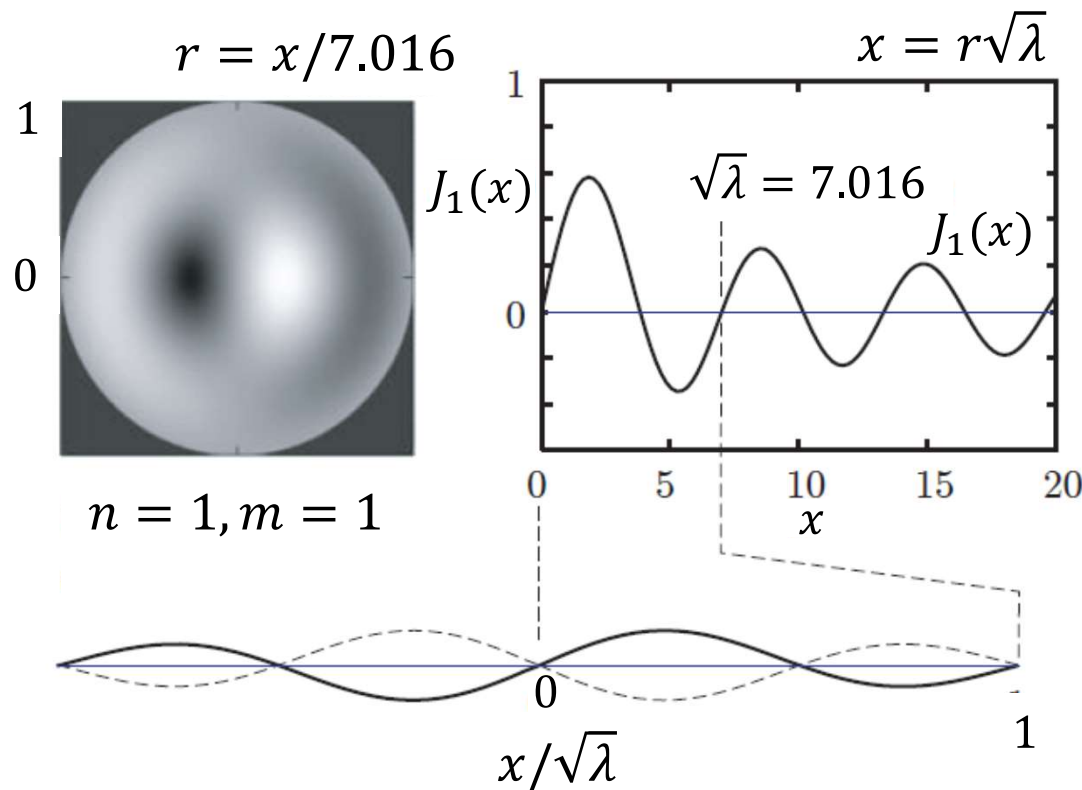
円形膜(太鼓)の振動

テキスト: p. 37

$$n = 1$$

$$\frac{\partial^2 y(x)}{\partial x^2} + \frac{1}{x} \frac{\partial y(x)}{\partial x} + \left(1 - \frac{n^2}{x^2}\right) y(x) = 0$$

$$\Phi(\theta) = \cos n\theta, \sin n\theta$$



角振動数

$$\omega = \pm \sqrt{\lambda \frac{T}{\rho}}$$

$$= \pm 7.016 \sqrt{\frac{T}{\rho}}$$

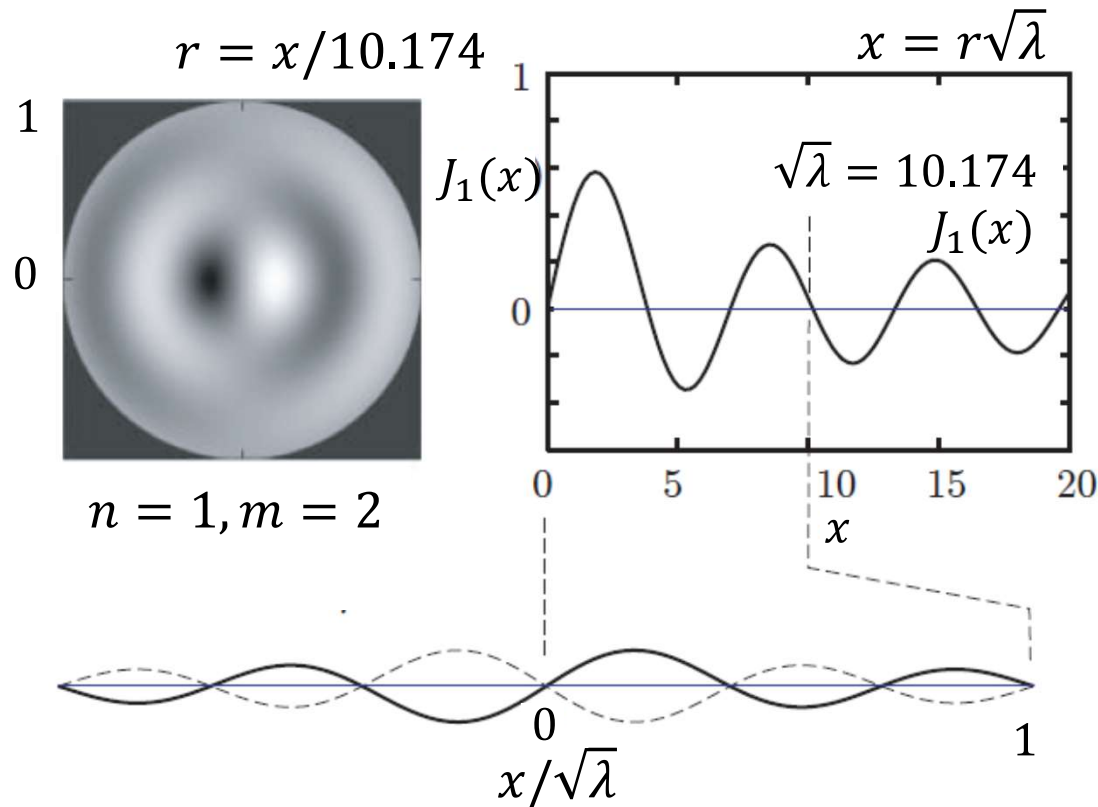
円形膜(太鼓)の振動

テキスト: p. 37

$$n = 1$$

$$\frac{\partial^2 y(x)}{\partial x^2} + \frac{1}{x} \frac{\partial y(x)}{\partial x} + \left(1 - \frac{n^2}{x^2}\right) y(x) = 0$$

$$\Phi(\theta) = \cos n\theta, \sin n\theta$$



角振動数

$$\omega = \pm \sqrt{\lambda \frac{T}{\rho}}$$

$$= \pm 10.174 \sqrt{\frac{T}{\rho}}$$

ベッセル関数の母関数

テキスト: p. 39

$$\exp \left\{ \frac{x}{2} (t - t^{-1}) \right\}$$

$$e^{\frac{x}{2}(t-t^{-1})} = J_0(x) + \{J_1(x)t + J_{-1}(x)t^{-1}\} + \cdots + \{J_n(x)t^n + J_{-n}(x)t^{-n}\} + \cdots$$

$$e^{\frac{x}{2}(t-t^{-1})} = e^{\frac{x}{2}t} \cdot e^{-\frac{x}{2}t^{-1}}$$

$$= \left(\sum_{n=0}^{\infty} \frac{x^n}{2^n n!} t^n \right) \left(\sum_{n=0}^{\infty} (-1)^n \frac{x^n}{2^n n!} t^{-n} \right)$$

$$= \left(1 + \frac{x}{2}t + \frac{x^2}{2^2 2!}t^2 + \frac{x^3}{2^3 3!}t^3 + \frac{x^4}{2^4 4!}t^4 + \cdots \right) \\ \times \left(1 - \frac{x}{2}t^{-1} + \frac{x^2}{2^2 2!}t^{-2} - \frac{x^3}{2^3 3!}t^{-3} + \frac{x^3}{2^3 3!}t^{-4} - \cdots \right)$$

ベッセル関数の母関数

テキスト: p. 39

$$\exp \left\{ \frac{x}{2} (t - t^{-1}) \right\} = \left(1 + \frac{x}{2} t + \frac{x^2}{2^2 2!} t^2 + \frac{x^3}{2^3 3!} t^3 + \frac{x^4}{2^4 4!} t^4 + \dots \right) \\ \times \left(1 - \frac{x}{2} t^{-1} + \frac{x^2}{2^2 2!} t^{-2} - \frac{x^3}{2^3 3!} t^{-3} + \frac{x^3}{2^3 3!} t^{-4} - \dots \right)$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n}{2^{2n} (n!)^2} x^{2n} + t \sum_{n=0}^{\infty} \frac{(-1)^n}{2^{2n+1} n! (n+1)!} x^{2n+1} + \dots \\ + t^{-1} \sum_{n=0}^{\infty} \frac{(-1)^1 (-1)^n}{2^{2n+1} n! (n+1)!} x^{2n+1} + \dots$$

$$= J_0(x) + J_1(x)t + J_{-1}(x)t^{-1} + \dots$$

$$J_n(x) = (-1)^n J_{-n}(x)$$