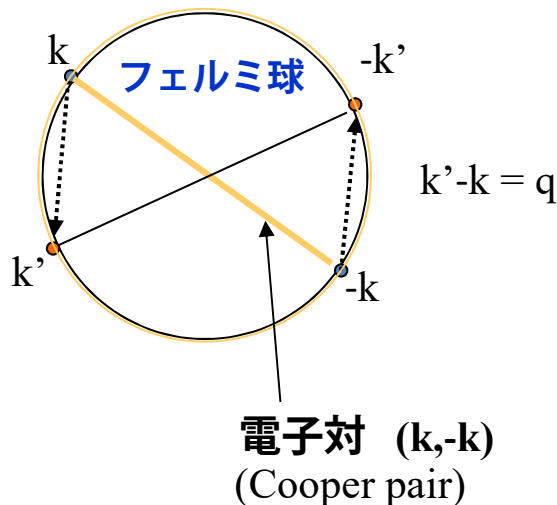


BCS 理論

1957 J.Bardeen, L.Cooper, J.R.Schrieffer

BCS 理論の基本概念

1. 電子対の形成によって引力エネルギーを効率的に確保
2. 電子対の重心をフェルミ球の中心に合わせることで散乱確率を最大化



有効相互作用

$$H_{eff} = - \sum_{kk'q\sigma\sigma'} \frac{\alpha_q^2}{\hbar\omega_q} c_{k+q\sigma}^+ c_{k'-q\sigma'}^+ c_{k'\sigma'} c_{k\sigma}$$



$$H_{eff} = \frac{1}{2} \sum_{kk'\sigma\sigma'} V(k', k) c_{k'\sigma}^+ c_{-k'\sigma'}^+ c_{-k\sigma} c_{k\sigma}$$

電子対のスピン状態

Singlet pair : singlet superconductor

$$H_{eff} = \frac{1}{2} \sum_{kk'} V(k', k) (c_{k'\downarrow}^+ c_{-k'\uparrow}^+ c_{-k\uparrow} c_{k\downarrow} + c_{k'\uparrow}^+ c_{-k'\downarrow}^+ c_{-k\downarrow} c_{k\uparrow})$$

Triplet pair : triplet superconductor

$$H_{eff} = \frac{1}{2} \sum_{kk'} V(k', k) (c_{k'\uparrow}^+ c_{-k'\uparrow}^+ c_{-k\uparrow} c_{k\uparrow} + c_{k'\downarrow}^+ c_{-k'\downarrow}^+ c_{-k\downarrow} c_{k\downarrow})$$

BCS 理論

シングレット超伝導体

$$H_{eff} = \frac{1}{2} \sum_{\mathbf{k}\mathbf{k}'} V(\mathbf{k}', \mathbf{k}) (c_{\mathbf{k}'\downarrow}^+ c_{-\mathbf{k}'\uparrow}^+ c_{-\mathbf{k}\uparrow} c_{\mathbf{k}\downarrow} + c_{\mathbf{k}'\uparrow}^+ c_{-\mathbf{k}'\downarrow}^+ c_{-\mathbf{k}\downarrow} c_{\mathbf{k}\uparrow})$$

等方的一重項対を仮定

$$V(\mathbf{k}', \mathbf{k}) = V(-\mathbf{k}', \mathbf{k}) = V(\mathbf{k}', -\mathbf{k}) = V(-\mathbf{k}', -\mathbf{k}) = V^*(\mathbf{k}, \mathbf{k}')$$

対状態の擬スピン表示

$$\begin{aligned} |0\rangle &\rightarrow |+_k\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} && \text{対なし} \\ &\quad \uparrow \text{擬スピン状態} \\ c_{-\mathbf{k}\downarrow}^+ c_{\mathbf{k}\uparrow}^+ |0\rangle &= s_k^- |+_k\rangle \rightarrow |-_k\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix} && \text{対あり} \\ &\quad \downarrow \text{擬スピン演算子} \\ s_k^+ |-_k\rangle &\rightarrow |+_k\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \end{aligned}$$

$$\begin{aligned} c_{\mathbf{k}\uparrow} c_{-\mathbf{k}\downarrow} &= s_k^+ = \frac{1}{2} (\sigma_k^x + i\sigma_k^y) \\ c_{-\mathbf{k}\downarrow} c_{\mathbf{k}\uparrow}^+ &= s_k^- = \frac{1}{2} (\sigma_k^x - i\sigma_k^y) \end{aligned}$$

$$(-c_{\mathbf{k}\uparrow}^+ c_{\mathbf{k}\uparrow} - c_{-\mathbf{k}\downarrow}^+ c_{-\mathbf{k}\downarrow} + 1) = \sigma_k^z$$

対あり: -1, 対なし: 1

$$s^x = \frac{1}{2} \sigma^x = \frac{1}{2} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad s^y = \frac{1}{2} \sigma^y = \frac{1}{2} \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad s^z = \frac{1}{2} \sigma^z = \frac{1}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$s^+ = s^x + is^y \quad s^- = s^x - is^y$$

$$H = \sum_{\mathbf{k}\sigma} (\varepsilon_{\mathbf{k}} - \varepsilon_F) c_{\mathbf{k}\sigma}^+ c_{\mathbf{k}\sigma} + \sum_{\mathbf{k}\mathbf{k}'} V(\mathbf{k}', \mathbf{k}) c_{-\mathbf{k}'\downarrow}^+ c_{\mathbf{k}'\uparrow}^+ c_{\mathbf{k}\uparrow} c_{-\mathbf{k}\downarrow}$$



$$H = -\sum_{\mathbf{k}} (\varepsilon_{\mathbf{k}} - \varepsilon_F) \sigma_k^z + \sum_{\mathbf{k}\mathbf{k}'} V(\mathbf{k}', \mathbf{k}) \frac{1}{4} (\sigma_k^x \sigma_{\mathbf{k}'}^x + \sigma_k^y \sigma_{\mathbf{k}'}^y)$$

$$s_k^+ s_{\mathbf{k}'}^- = \frac{1}{2} (s_k^+ s_{\mathbf{k}'}^- + s_{\mathbf{k}'}^- s_k^+) \rightarrow \frac{1}{2} (s_k^+ s_{\mathbf{k}'}^- + s_{\mathbf{k}'}^- s_k^+) = s_k^x s_{\mathbf{k}'}^x + s_k^y s_{\mathbf{k}'}^y$$

$$\mathbf{k}' \approx \mathbf{k}$$

BCS 理論

$$H = -\sum_{\mathbf{k}} (\varepsilon_{\mathbf{k}} - \varepsilon_F) \sigma_{\mathbf{k}}^z + \sum_{\mathbf{k}\mathbf{k}'} V(\mathbf{k}', \mathbf{k}) \frac{1}{4} (\sigma_{\mathbf{k}'}^x \sigma_{\mathbf{k}}^x + \sigma_{\mathbf{k}'}^y \sigma_{\mathbf{k}}^y)$$

Zeeman 項
z-方向
XY 相互作用
xy-面内
磁場中 XY 模型

平均場近似

$$\begin{aligned} \sigma_{\mathbf{k}'}^x \sigma_{\mathbf{k}}^x &= \left\{ \langle \sigma_{\mathbf{k}'}^x \rangle + (\sigma_{\mathbf{k}'}^x - \langle \sigma_{\mathbf{k}'}^x \rangle) \right\} \left\{ \langle \sigma_{\mathbf{k}}^x \rangle + (\sigma_{\mathbf{k}}^x - \langle \sigma_{\mathbf{k}}^x \rangle) \right\} && \text{2 次の揺らぎを無視} \\ &= \langle \sigma_{\mathbf{k}'}^x \rangle \langle \sigma_{\mathbf{k}}^x \rangle + \langle \sigma_{\mathbf{k}'}^x \rangle (\sigma_{\mathbf{k}}^x - \langle \sigma_{\mathbf{k}}^x \rangle) + (\sigma_{\mathbf{k}'}^x - \langle \sigma_{\mathbf{k}'}^x \rangle) \langle \sigma_{\mathbf{k}}^x \rangle + (\sigma_{\mathbf{k}'}^x - \langle \sigma_{\mathbf{k}'}^x \rangle) (\sigma_{\mathbf{k}}^x - \langle \sigma_{\mathbf{k}}^x \rangle) \\ &\approx \langle \sigma_{\mathbf{k}'}^x \rangle \sigma_{\mathbf{k}}^x + \sigma_{\mathbf{k}'}^x \langle \sigma_{\mathbf{k}}^x \rangle - \langle \sigma_{\mathbf{k}'}^x \rangle \langle \sigma_{\mathbf{k}}^x \rangle && \delta\sigma_{\mathbf{k}'}^x \quad \delta\sigma_{\mathbf{k}}^x \end{aligned}$$

$$H = -\sum_{\mathbf{k}} (\varepsilon_{\mathbf{k}} - \varepsilon_F) \sigma_{\mathbf{k}}^z + \sum_{\mathbf{k}\mathbf{k}'} V(\mathbf{k}', \mathbf{k}) \frac{1}{4} \left[2 \langle \sigma_{\mathbf{k}'}^x \rangle \sigma_{\mathbf{k}}^x + 2 \langle \sigma_{\mathbf{k}'}^y \rangle \sigma_{\mathbf{k}}^y - \langle \sigma_{\mathbf{k}'}^x \rangle \langle \sigma_{\mathbf{k}}^x \rangle - \langle \sigma_{\mathbf{k}'}^y \rangle \langle \sigma_{\mathbf{k}}^y \rangle \right]$$

$$\Delta_{\mathbf{k}} = -\sum_{\mathbf{k}'} V(\mathbf{k}', \mathbf{k}) \langle c_{\mathbf{k}\uparrow} c_{-\mathbf{k}\downarrow} \rangle \quad \xi_{\mathbf{k}} = \varepsilon_{\mathbf{k}} - \varepsilon_F$$

$$H = \sum_{\mathbf{k}\sigma} \xi_{\mathbf{k}} c_{\mathbf{k}\sigma}^{\dagger} c_{\mathbf{k}\sigma} - \sum_{\mathbf{k}} \left(\Delta_{\mathbf{k}} c_{-\mathbf{k}\downarrow}^{\dagger} c_{\mathbf{k}\uparrow}^{\dagger} + \Delta_{\mathbf{k}}^* c_{\mathbf{k}\uparrow} c_{-\mathbf{k}\downarrow} \right) + \sum_{\mathbf{k}} \Delta_{\mathbf{k}} \langle c_{-\mathbf{k}\downarrow}^{\dagger} c_{\mathbf{k}\uparrow}^{\dagger} \rangle$$