

BCS 理論

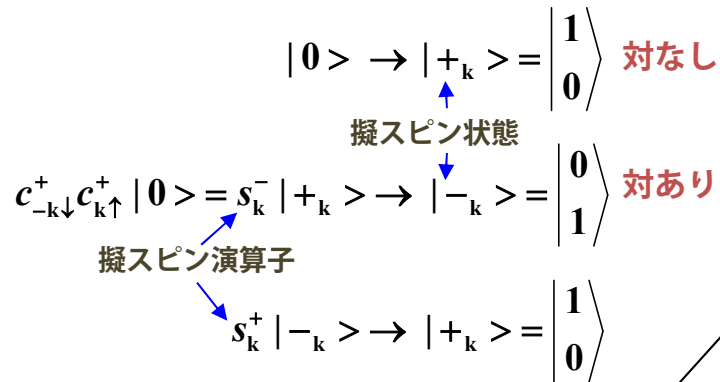
シングレット超伝導体

$$H_{eff} = \frac{1}{2} \sum_{\mathbf{k}\mathbf{k}'} V(\mathbf{k}', \mathbf{k}) (c_{\mathbf{k}'\downarrow}^+ c_{-\mathbf{k}'\uparrow}^+ c_{-\mathbf{k}\uparrow} c_{\mathbf{k}\downarrow} + c_{\mathbf{k}'\uparrow}^+ c_{-\mathbf{k}'\downarrow}^+ c_{-\mathbf{k}\downarrow} c_{\mathbf{k}\uparrow})$$

等方的一重項対を仮定

$$V(\mathbf{k}', \mathbf{k}) = V(-\mathbf{k}', \mathbf{k}) = V(\mathbf{k}', -\mathbf{k}) = V(-\mathbf{k}', -\mathbf{k}) = V^*(\mathbf{k}, \mathbf{k}')$$

対状態の擬スピン表示



$$c_{k\uparrow} c_{-k\downarrow} = s_k^+ = \frac{1}{2} (\sigma_k^x + i\sigma_k^y)$$

$$c_{-k\downarrow} c_{k\uparrow} = s_k^- = \frac{1}{2} (\sigma_k^x - i\sigma_k^y)$$

$$(-c_{k\uparrow}^+ c_{k\uparrow} - c_{-k\downarrow}^+ c_{-k\downarrow} + 1) = \sigma_k^z$$

電子対あり: -1, 電子対なし: 1

$$s^x = \frac{1}{2} \sigma^x = \frac{1}{2} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad s^y = \frac{1}{2} \sigma^y = \frac{1}{2} \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad s^z = \frac{1}{2} \sigma^z = \frac{1}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$s^+ = s^x + is^y \quad s^- = s^x - is^y$$

$$H = \sum_{\mathbf{k}\sigma} (\varepsilon_{\mathbf{k}} - \varepsilon_F) c_{\mathbf{k}\sigma}^+ c_{\mathbf{k}\sigma} + \sum_{\mathbf{k}\mathbf{k}'} V(\mathbf{k}', \mathbf{k}) c_{-\mathbf{k}'\downarrow}^+ c_{\mathbf{k}'\uparrow}^+ c_{\mathbf{k}\uparrow} c_{-\mathbf{k}\downarrow}$$



$$H = -\sum_{\mathbf{k}} (\varepsilon_{\mathbf{k}} - \varepsilon_F) \sigma_k^z + \sum_{\mathbf{k}\mathbf{k}'} V(\mathbf{k}', \mathbf{k}) \frac{1}{4} (\sigma_k^x \sigma_{k'}^x + \sigma_k^y \sigma_{k'}^y)$$

$$s_k^+ s_k^- = \frac{1}{2} (s_k^+ s_k^- + s_k^- s_k^+) \rightarrow \frac{1}{2} (s_k^+ s_k^- + s_k^- s_k^+) = s_k^x s_k^x + s_k^y s_k^y$$

$\mathbf{k}' \rightleftharpoons \mathbf{k}$

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$$H = -\sum_{\mathbf{k}} (\varepsilon_{\mathbf{k}} - \varepsilon_F) \sigma_{\mathbf{k}}^z + \sum_{\mathbf{k}\mathbf{k}'} V(\mathbf{k}', \mathbf{k}) \frac{1}{4} (\sigma_{\mathbf{k}'}^x \sigma_{\mathbf{k}}^x + \sigma_{\mathbf{k}'}^y \sigma_{\mathbf{k}}^y)$$

Zeeman 項
z-方向
XY 相互作用
xy-面内
磁場中 XY 模型

平均場近似

$$\begin{aligned} \sigma_{\mathbf{k}'}^x \sigma_{\mathbf{k}}^x &= \left\{ \langle \sigma_{\mathbf{k}'}^x \rangle + (\sigma_{\mathbf{k}'}^x - \langle \sigma_{\mathbf{k}'}^x \rangle) \right\} \left\{ \langle \sigma_{\mathbf{k}}^x \rangle + (\sigma_{\mathbf{k}}^x - \langle \sigma_{\mathbf{k}}^x \rangle) \right\} && \text{2 次の揺らぎを無視} \\ &= \langle \sigma_{\mathbf{k}'}^x \rangle \langle \sigma_{\mathbf{k}}^x \rangle + \langle \sigma_{\mathbf{k}'}^x \rangle (\sigma_{\mathbf{k}}^x - \langle \sigma_{\mathbf{k}}^x \rangle) + (\sigma_{\mathbf{k}'}^x - \langle \sigma_{\mathbf{k}'}^x \rangle) \langle \sigma_{\mathbf{k}}^x \rangle + (\sigma_{\mathbf{k}'}^x - \langle \sigma_{\mathbf{k}'}^x \rangle) (\sigma_{\mathbf{k}}^x - \langle \sigma_{\mathbf{k}}^x \rangle) \\ &\approx \langle \sigma_{\mathbf{k}'}^x \rangle \sigma_{\mathbf{k}}^x + \sigma_{\mathbf{k}'}^x \langle \sigma_{\mathbf{k}}^x \rangle - \langle \sigma_{\mathbf{k}'}^x \rangle \langle \sigma_{\mathbf{k}}^x \rangle && \delta\sigma_{\mathbf{k}'}^x \quad \delta\sigma_{\mathbf{k}}^x \end{aligned}$$

$$H = -\sum_{\mathbf{k}} (\varepsilon_{\mathbf{k}} - \varepsilon_F) \sigma_{\mathbf{k}}^z + \sum_{\mathbf{k}\mathbf{k}'} V(\mathbf{k}', \mathbf{k}) \frac{1}{4} \left[2 \langle \sigma_{\mathbf{k}'}^x \rangle \sigma_{\mathbf{k}}^x + 2 \langle \sigma_{\mathbf{k}'}^y \rangle \sigma_{\mathbf{k}}^y - \langle \sigma_{\mathbf{k}'}^x \rangle \langle \sigma_{\mathbf{k}}^x \rangle - \langle \sigma_{\mathbf{k}'}^y \rangle \langle \sigma_{\mathbf{k}}^y \rangle \right]$$

$$\Delta_{\mathbf{k}} = -\sum_{\mathbf{k}'} V(\mathbf{k}', \mathbf{k}) \langle c_{\mathbf{k}\uparrow} c_{-\mathbf{k}\downarrow} \rangle \quad \xi_{\mathbf{k}} = \varepsilon_{\mathbf{k}} - \varepsilon_F$$

$$H = \sum_{\mathbf{k}\sigma} \xi_{\mathbf{k}} c_{\mathbf{k}\sigma}^{\dagger} c_{\mathbf{k}\sigma} - \sum_{\mathbf{k}} \left(\Delta_{\mathbf{k}} c_{-\mathbf{k}\downarrow}^{\dagger} c_{\mathbf{k}\uparrow}^{\dagger} + \Delta_{\mathbf{k}}^* c_{\mathbf{k}\uparrow} c_{-\mathbf{k}\downarrow} \right) + \sum_{\mathbf{k}} \Delta_{\mathbf{k}} \langle c_{-\mathbf{k}\downarrow}^{\dagger} c_{\mathbf{k}\uparrow}^{\dagger} \rangle$$

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$$\begin{aligned} \text{基底状態 } |G\rangle &= \prod_k \alpha_{-k\downarrow} \alpha_{k\uparrow} |0\rangle \\ &= \prod_k (u_k c_{-k\downarrow} + v_k c_{k\uparrow}^+) (u_k c_{k\uparrow} - v_k c_{-k\downarrow}^+) |0\rangle \\ &= \prod_k -v_k (u_k + v_k c_{k\uparrow}^+ c_{-k\downarrow}^+) |0\rangle \end{aligned}$$

$$H = \sum_{k\sigma} \xi_k c_{k\sigma}^+ c_{k\sigma} - \sum_k (\Delta_k c_{-k\downarrow}^+ c_{k\uparrow}^+ + \Delta_k^* c_{k\uparrow} c_{-k\downarrow}) + \sum_k \Delta_k \langle c_{-k\downarrow}^+ c_{k\uparrow}^+ \rangle$$

Bogoliubov 変換

$$\begin{aligned} \alpha_{k\uparrow} &= u_k c_{k\uparrow} - v_k c_{-k\downarrow}^+ \\ \alpha_{-k\downarrow}^+ &= u_k c_{-k\downarrow}^+ + v_k^* c_{k\uparrow} \end{aligned}$$

逆変換

$$\begin{aligned} c_{k\uparrow} &= u_k \alpha_{k\uparrow} + v_k \alpha_{-k\downarrow}^+ \\ c_{-k\downarrow}^+ &= u_k \alpha_{-k\downarrow}^+ - v_k^* \alpha_{k\uparrow} \end{aligned}$$

$$[\alpha_{k\uparrow}, \alpha_{k\uparrow}^+]_+ = \alpha_{k\uparrow} \alpha_{k\uparrow}^+ + \alpha_{k\uparrow}^+ \alpha_{k\uparrow} = 1$$

Bogoliubov 粒子 α_k^+ はフェルミオンとして振る舞う

u_k : 実数

$$v_k = |v_k| e^{i\phi_k}$$

$$u_k^2 + |v_k|^2 = 1$$

$$u_k^2 = \frac{1}{2} \left(1 + \frac{\xi_k}{E_k} \right), \quad |v_k|^2 = \frac{1}{2} \left(1 - \frac{\xi_k}{E_k} \right), \quad E_k = \sqrt{\xi_k^2 + |\Delta_k|^2}, \quad \Delta_k = |\Delta_k| e^{i\phi_k}$$

$$\alpha_{-k\downarrow}^+ \alpha_{k\uparrow}^+, \quad \alpha_{-k\downarrow} \alpha_{k\uparrow} \text{ を消す条件} \longrightarrow 2\xi_k u_k v_k + \Delta_k^* v_k^2 - \Delta_k u_k^2 = 0$$

$$H = E_g + \sum_k E_k (\alpha_{k\uparrow}^+ \alpha_{k\uparrow} + \alpha_{-k\downarrow}^+ \alpha_{-k\downarrow})$$

$$E_g = \sum_k (2\xi_k |v_k|^2 + 2\Delta_k u_k v_k^* + \Delta_k \langle c_{-k\downarrow}^+ c_{k\uparrow}^+ \rangle)$$

